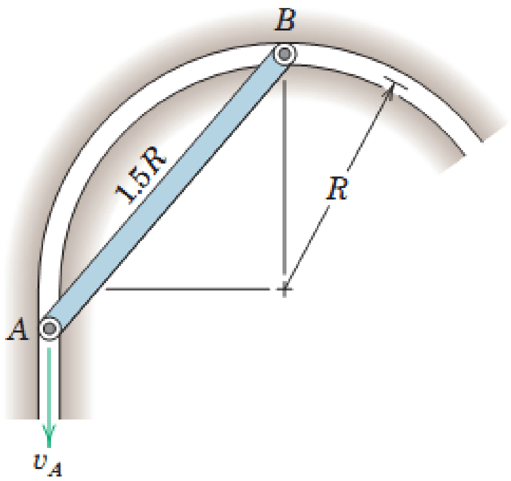
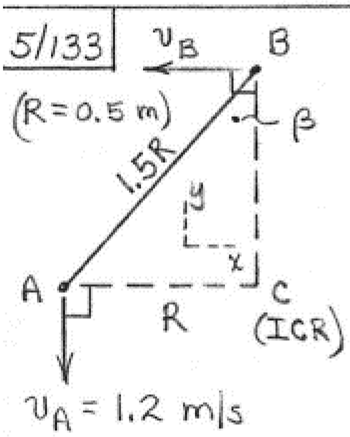
ME 206 – DYNAMICS – SPRING 2017
STUDY PROBLEMS-9 (RIGID BODY KINEMATICS-SECTION 5.6)

PROBLEM 5/133

The end rollers of bar AB are constrained to the slot shown. If roller A has a downward velocity of 1.2 m/s and this speed is constant over a small motion interval, determine the tangential acceleration of roller B as it passes the topmost position. The value of R is 0.5 m.



5/133 (R = 0.5 m)



$\beta = \sin^{-1}\dfrac{R}{1.5R} = 41.8°$

$\overline{BC} = 1.5R\cos\beta = 1.118R$

$= 0.559$ m

$\omega = \dfrac{v_A}{R} = \dfrac{1.2}{0.5} = 2.4$ rad/s (CCW)

$v_B = \overline{BC}\,\omega = 0.559(2.4)$

$= 1.342$ m/s (left)

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A} = \underline{a}_A^{\nearrow 0} + \underline{\alpha} \times \underline{r}_{B/A} - \omega^2 \underline{r}_{B/A}$

$-\dfrac{1.342^2}{0.5}\underline{j} + (a_B)_t\,\underline{i} = \alpha\underline{k} \times (0.5\underline{i} + 0.559\underline{j})$

$-2.4^2(0.5\underline{i} + 0.559\underline{j})$

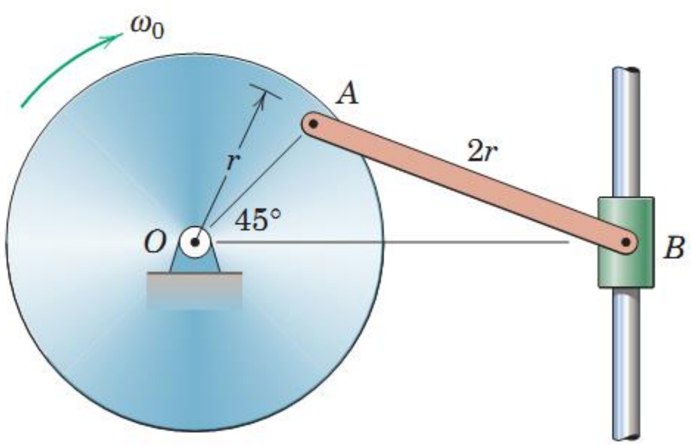
$\underline{i}: \quad (a_B)_t = -0.559\alpha - 2.88$

$\underline{j}: \quad -3.6 = 0.5\alpha - 3.22$

$\alpha = -0.760$ rad/s^2 (CW), $\quad (a_B)_t = -2.46\ \dfrac{m}{s^2}$ (left)

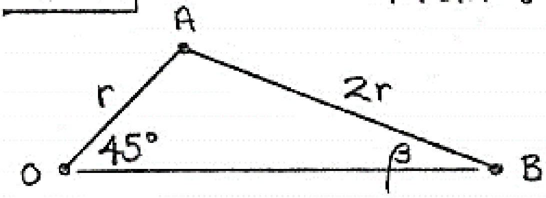
PROBLEM 5/135

The mechanism of Prob. 5/76 is repeated here. The angular velocity ω_0 of the disk is constant. For the instant represented, determine the angular acceleration α_{AB} of link AB and the acceleration a_B of collar B. Assume the quantities ω_0 and r to be known.



5/135

From the solution to Prob. 5/76,



$\beta = 20.7°$

$\omega_{AB} = \omega_0$ CW

$$\underline{a}_A = \underline{a}_B + \underline{\alpha} \times \underline{r}_{A/B} - \omega^2 \underline{r}_{A/B}$$

$$-r\omega_0^2\left[\frac{\sqrt{2}}{2}\underline{i} + \frac{\sqrt{2}}{2}\underline{j}\right] = a_B\underline{j} + \alpha\underline{k} \times 2r[-\cos\beta\,\underline{i} + \sin\beta\,\underline{j}] - \omega_0^2(2r)[-\cos\beta\,\underline{i} + \sin\beta\,\underline{j}]$$

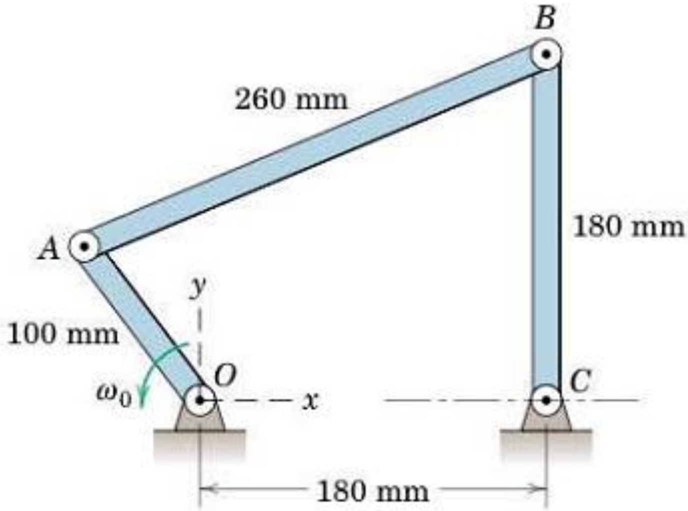
$$\begin{cases} \underline{i}: -r\omega_0^2\frac{\sqrt{2}}{2} = -2r\alpha\sin\beta + 2r\omega_0^2\cos\beta \\ \underline{j}: -r\omega_0^2\frac{\sqrt{2}}{2} = a_B - 2r\alpha\cos\beta - 2r\omega_0^2\sin\beta \end{cases}$$

With the above value of β: $\alpha = \alpha_{AB} = 3.64\omega_0^2$ (CCW)

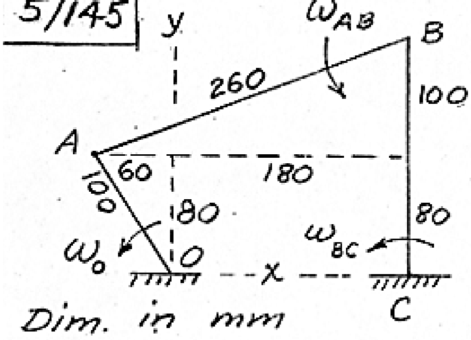
$a_B = +6.82 r\omega_0^2$ (↑)

PROBLEM 5/145 (6th Ed.)

The linkage of Prob. 5/88 is shown again here. If *OA* has a constant counterclockwise angular velocity $\omega_0 = 10$ rad/s, calculate the angular acceleration of link *AB* for the position where the coordinates of *A* are $x = -60$ mm and $y = 80$ mm. Link *BC* is vertical for this position. Solve by vector algebra. (Use the results of Prob. 5/88 for the angular velocities of *AB* and *BC*, which are $\boldsymbol{\omega}_{BC} = 5.83\mathbf{k}$ rad/s and $\boldsymbol{\omega}_{AB} = 2.5\mathbf{k}$ rad/s.)



5/145



$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$\underline{a}_B = \underline{\omega}_{BC}\times(\underline{\omega}_{BC}\times \underline{r}_{B/C}) + \underline{\alpha}_{BC}\times \underline{r}_{B/C}$

$= 5.83\underline{k}\times(5.83\underline{k}\times 0.18\underline{j})$

$+ \alpha_{BC}\underline{k}\times 0.18\underline{j}$ m/s²

$= -6.125\underline{j} - 0.18\alpha_{BC}\underline{i}$ m/s²

$\underline{a}_A = \underline{\omega}_0\times(\underline{\omega}_0\times \underline{r}_{A/O}) = 10\underline{k}\times(10\underline{k}\times[-0.06\underline{i}+0.08\underline{j}])$

$= 6\underline{i} - 8\underline{j}$ m/s² $\quad (\underline{\alpha}_{OA} = 0)$

$(a_{B/A})_n = \underline{\omega}_{AB}\times(\underline{\omega}_{AB}\times \underline{r}_{B/A}) = 2.5\underline{k}\times(2.5\underline{k}\times[0.24\underline{i}+0.1\underline{j}])$

$= -1.5\underline{i} - 0.625\underline{j}$ m/s²

$(a_{B/A})_t = \alpha_{AB}\underline{k}\times(0.24\underline{i}+0.1\underline{j}) = -0.1\alpha_{AB}\underline{i} + 0.24\alpha_{AB}\underline{j}$

Substitute in accel. equation & equate coefficients & set

$-0.18\alpha_{BC} = 6 - 1.5 - 0.1\alpha_{AB}$

$-6.125 = -8 - 0.625 + 0.24\alpha_{AB}$

Sol. is

$\underline{\alpha}_{AB} = \underline{10.42\underline{k}\text{ rad/s}^2}$

$(\underline{\alpha}_{BC} = -19.21\underline{k}\text{ rad/s}^2)$