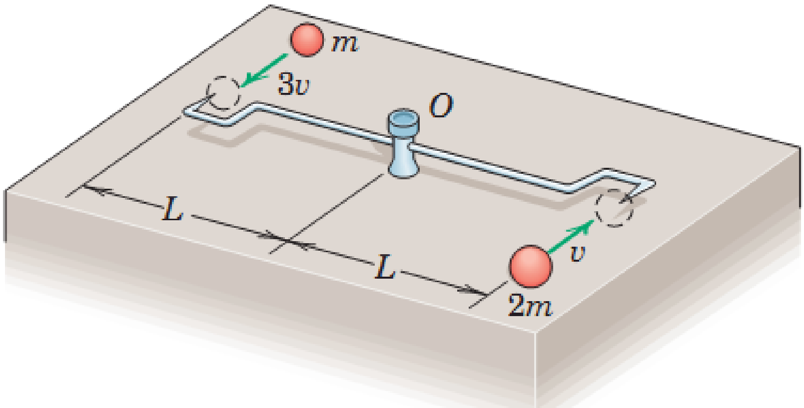
ME 206 – DYNAMICS – SPRING 2017
STUDY PROBLEMS-7b
(PART C. ANGULAR IMPULSE-MOMENTUM)

PROBLEM 3/220

The small spheres, which have the masses and initial velocities shown in the figure, strike and become attached to the spiked ends of the rod, which is freely pivoted at O and is initially at rest. Determine the angular velocity ω of the assembly after impact. Neglect the mass of the rod.



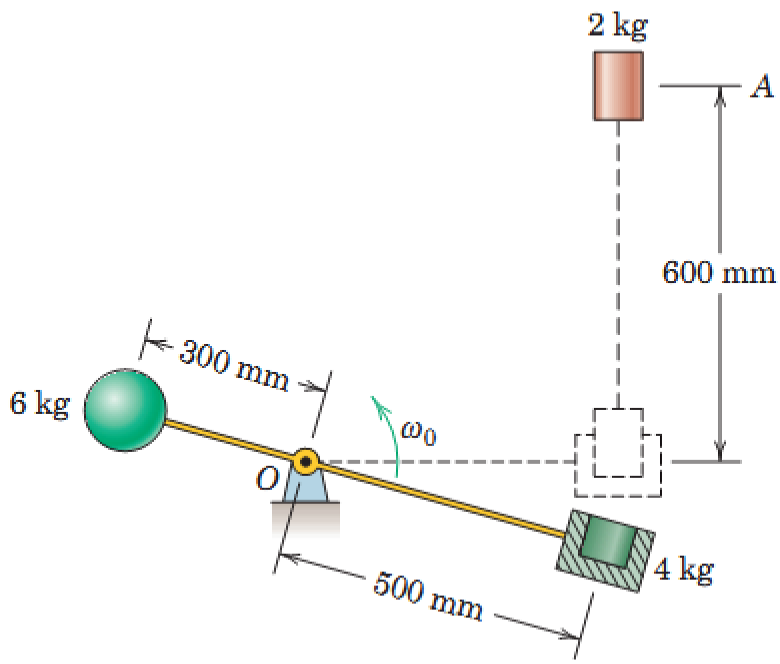
Angular momentum about O is conserved:

$H_{O_1} = H_{O_2}$: $3mv(L) + 2mv(L) = 3mL^2\omega$

$$\omega = \frac{5}{3}\frac{v}{L}$$

PROBLEM 3/227

The 6-kg sphere and 4-kg block (shown in section) are secured to the arm of negligible mass which rotates in the vertical plane about a horizontal axis at O. The 2-kg plug is released from rest at A and falls into the recess in the block when the arm has reached the horizontal position. An instant before engagement, the arm has an angular velocity $\omega_0 = 2$ rad/s. Determine the angular velocity ω of the arm immediately after the plug has wedged itself in the block.



3/227 Velocity of plug upon impact is

$$v = \sqrt{2gh} = \sqrt{2(9.81)(0.6)} = 3.43 \text{ m/s}$$

For system, $\Delta H_O = 0$. Take C.W. positive

Initial $H_O = -4(0.5)^2(2) - 6(0.3)^2(2) + 2(3.43)(0.5)$

$= -2 - 1.08 + 3.43 = 0.351$ N·m·s

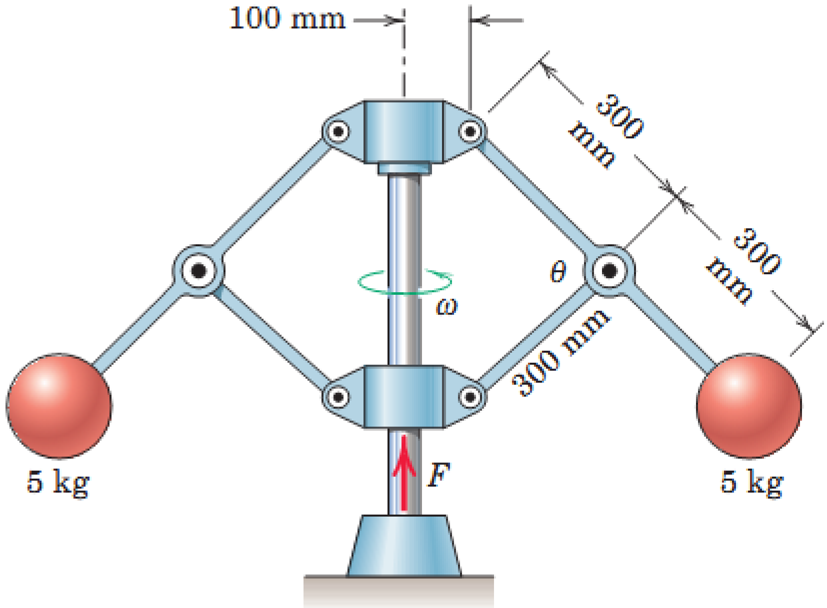
Final $H_O = [(4+2)(0.5)^2 + 6(0.3)^2]\omega$

$= 2.04\omega$

So $0.351 = 2.04\omega$, $\omega = 0.1721$ rad/s CW

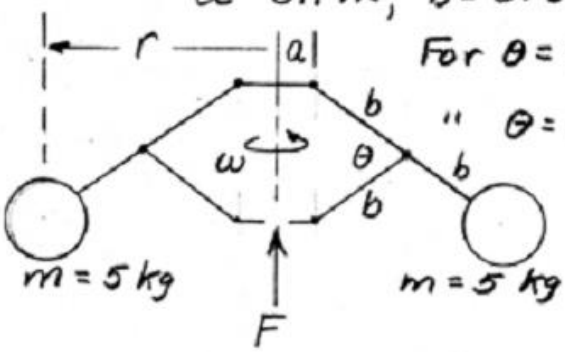
PROBLEM 3/238

The assembly of two 5-kg spheres is rotating freely about the vertical axis at 40 rev/min with $\theta = 90°$. If the force F which maintains the given position is increased to raise the base collar and reduce θ to 60°, determine the new angular velocity ω. Also determine the work U done by F in changing the configuration of the system. Assume that the mass of the arms and collars is negligible.



$\omega_0 = 40(2\pi)/60 = 4.19$ rad/s

$a = 0.1$ m, $b = 0.3$ m



For $\theta = 90°$, $r_0 = 0.1 + 2(0.3)\cos 45° = 0.524$ m

" $\theta = 60°$, $r = 0.1 + 2(0.3)\cos 30° = 0.620$ m

$\Delta H = 0$; $2mr_0^2\omega_0 - 2mr^2\omega = 0$

$\omega = \frac{r_0^2}{r^2}\omega_0 = \left(\frac{0.524}{0.620}\right)^2(4.19)$

$= \underline{3.00 \text{ rad/s}}$

(or $\frac{3.00}{2\pi}60 = 28.6$ rev/min)

$U = \Delta T + \Delta V_g = 2\left(\frac{1}{2}m\right)\left(r^2\omega^2 - r_0^2\omega_0^2\right) + 2mg\,\Delta h$

where $\Delta h = 2b(\sin 45° - \sin 30°)$

$= 2(0.3)(0.7071 - 0.5) = 0.1243$ m

$U = 5\left([0.620 \times 3.00]^2 - [0.524 \times 4.19]^2\right) + 2(5)(9.81)(0.1243)$

$= -6.850 + 12.190 = \underline{5.34 \text{ J}}$