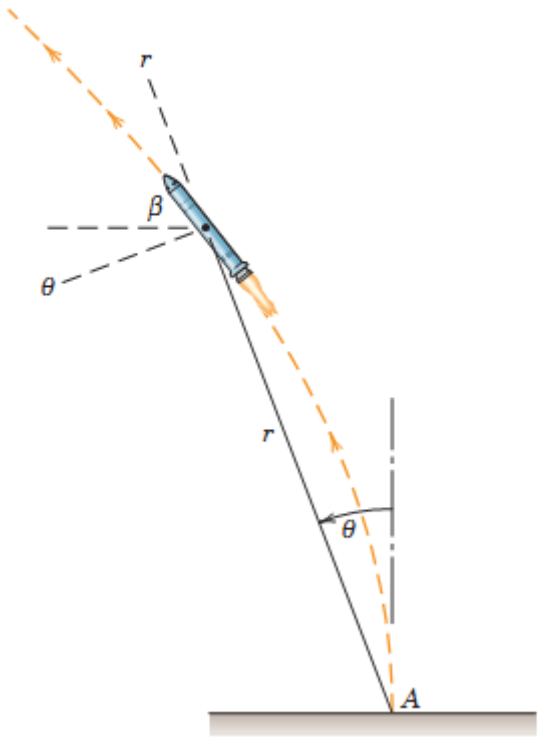
ME 206 – DYNAMICS – SPRING 2017
STUDY PROBLEMS-3 (PARTICLE KINEMATICS, SECTION 2.6)

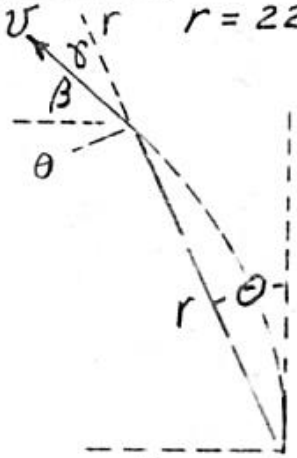
PROBLEM 2/151



A rocket is tracked by radar from its launching point A. When it is 10 seconds into its flight, the following radar measurements are recorded: $r = 2200$ m, $\dot{r} = 500$ m/s, $\ddot{r} = 4.66$ m/s^2, $\theta = 22°$, $\dot{\theta} = 0.0788$ rad/s, and $\ddot{\theta} = -0.0341$ rad/s^2. For this instant determine the angle β between the horizontal and the direction of the trajectory of the rocket and find the magnitudes of its velocity $\mathbf{v}$ and acceleration $\mathbf{a}$.

2/151 $\theta = 22°$, $\dot{\theta} = 0.0788$ rad/s, $\ddot{\theta} = -0.0341$ rad/s^2

$r = 2200$ m, $\dot{r} = 500$ m/s, $\ddot{r} = 4.66$ m/s^2



$v_r = \dot{r}$, $v_\theta = r\dot{\theta} = 2200(0.0788) = 173.4$ m/s

$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(500)^2 + (173.4)^2} = \underline{529 \text{ m/s}}$

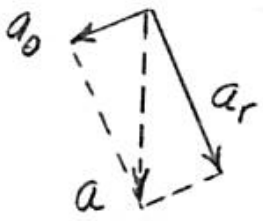
$\gamma = \tan^{-1} v_\theta / v_r = \tan^{-1} \frac{173.4}{500} = 19.12°$

$\beta = 90 - \gamma - \theta = 90 - 19.12 - 22 = \underline{48.9°}$

$a_r = \ddot{r} - r\dot{\theta}^2 = 4.66 - 2200(0.0788)^2 = -9.00$ m/s^2

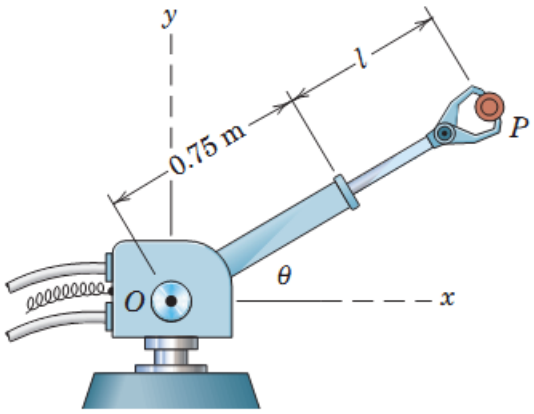
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2200(-0.0341) + 2(500)(0.0788) = 3.78$ m/s^2

$a = \sqrt{a_\theta^2 + a_r^2} = \sqrt{(3.78)^2 + (9.00)^2} = \underline{9.76 \text{ m/s}^2}$



PROBLEM 2/157

The robot arm is elevating and extending simultaneously. At a given instant, $\theta = 30°$, $\dot{\theta} = 10$ deg/s = constant, $l = 0.5$ m, $\dot{l} = 0.2$ m/s, and $\ddot{l} = -0.3$ m/s^2. Compute the magnitudes of the velocity **v** and acceleration **a** of the gripped part P. In addition, express **v** and **a** in terms of the unit vectors **i** and **j**.



$$\begin{cases} r = 0.75 + 0.5 = 1.25\ \text{m} & \theta = 30° \\ \dot{r} = 0.2\ \text{m/s} & \dot{\theta} = 0.1745\ \text{rad/s} \\ \ddot{r} = -0.3\ \text{m/s}^2 & \ddot{\theta} = 0 \end{cases}$$

$$\underline{v} = v_r \underline{e}_r + v_\theta \underline{e}_\theta = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$$

$$= 0.2\underline{e}_r + 1.25(0.1745)\underline{e}_\theta = 0.2\underline{e}_r + 0.218\underline{e}_\theta\ \text{m/s}$$

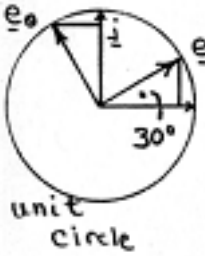
$$v = \sqrt{v_r^2 + v_\theta^2} = \underline{0.296\ \text{m/s}}$$

$$\underline{a} = a_r \underline{e}_r + a_\theta \underline{e}_\theta = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$$

$$= [-0.3 - 1.25(0.1745)^2]\underline{e}_r + [1.25(0) + 2(0.2)(0.1745)]\underline{e}_\theta$$

$$= -0.338\underline{e}_r + 0.0698\underline{e}_\theta\ \text{m/s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \underline{0.345\ \text{m/s}^2}$$



$$\underline{e}_r = \underline{i}\cos 30° + \underline{j}\sin 30°$$

$$\underline{e}_\theta = -\underline{i}\sin 30° + \underline{j}\cos 30°$$

$$\underline{v} = 0.2[\underline{i}\,c30° + \underline{j}\,s30°] + 0.218[-\underline{i}\,s30° + \underline{j}\,c30°]$$

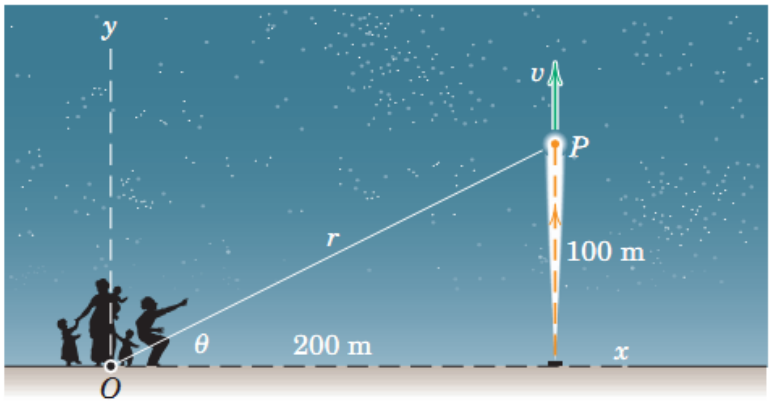
$$= \underline{0.064\underline{i} + 0.289\underline{j}\ \text{m/s}}$$

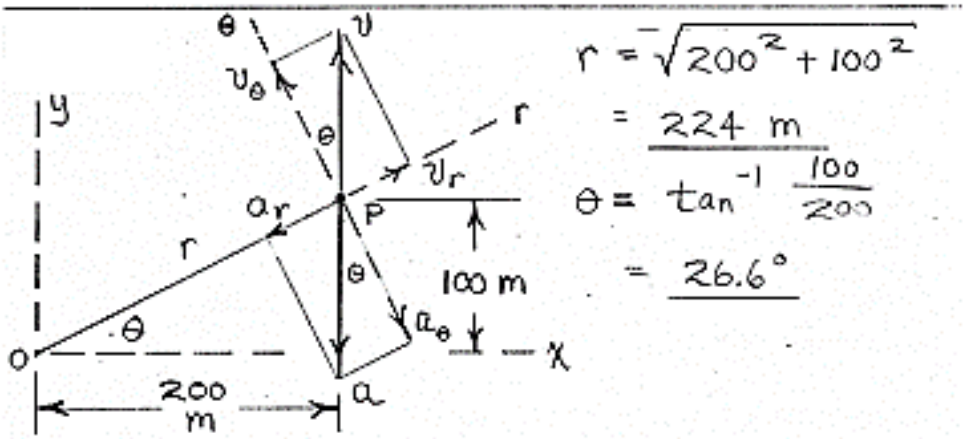
$$\underline{a} = -0.338[\underline{i}\,c30° + \underline{j}\,s30°] + 0.0698[-\underline{i}\,s30° + \underline{j}\,c30°]$$

$$= \underline{-0.328\,\underline{i} - 0.1086\underline{j}\ \text{m/s}^2}$$

PROBLEM 2/162

A fireworks shell P fired in a vertical trajectory has a y-acceleration given by $a_y = -g - kv^2$, where the latter term is due to aerodynamic drag. If the speed of the shell is 15 m/s at the instant shown, determine the corresponding values of r, $\dot{r}$, $\ddot{r}$, θ, $\dot{\theta}$, and $\ddot{\theta}$. The drag parameter k has a constant value of $0.01\ \text{m}^{-1}$.





$r = \sqrt{200^2 + 100^2}$

$= 224$ m

$\theta = \tan^{-1}\frac{100}{200}$

$= 26.6°$

$v_r = \dot{r} = v \sin\theta = 15 \sin 26.6° = 6.71$ m/s

$v_\theta = r\dot{\theta}$: $15 \cos 26.6° = 224\,\dot{\theta}$, $\dot{\theta} = 0.06$ rad/s

$a = -g - kv^2 = -9.81 - 0.01(15)^2 = -12.06$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2$: $-12.06 \sin 26.6° = \ddot{r} - 224(0.06)^2$

$\ddot{r} = -4.59$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$: $-12.06 \cos 26.6° = 224\ddot{\theta} + 2(6.71)(0.06)$

$\ddot{\theta} = -0.0518$ rad/s^2