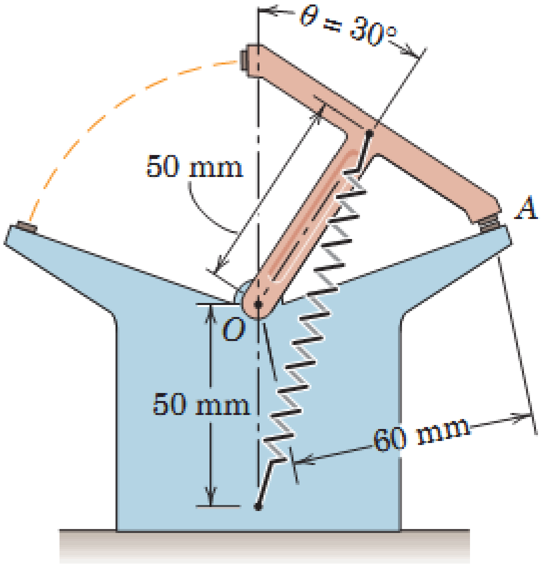
ME 206 – DYNAMICS – SPRING 2017
STUDY PROBLEMS-12
(PLANE KINETICS OF RIGID BODIES: WORK-ENERGY EQUATION)

PROBLEM 6/121 (WORK-ENERGY)

Specify the unstretched length l_0 of the spring of stiffness $k = 1400$ N/m which will result in a velocity of 0.25 m/s for the contact at A if the toggle is given a slight nudge from its null position at $\theta = 0$. The toggle has a mass of 1.5 kg and a radius of gyration about O of 55 mm. Motion occurs in the horizontal plane.



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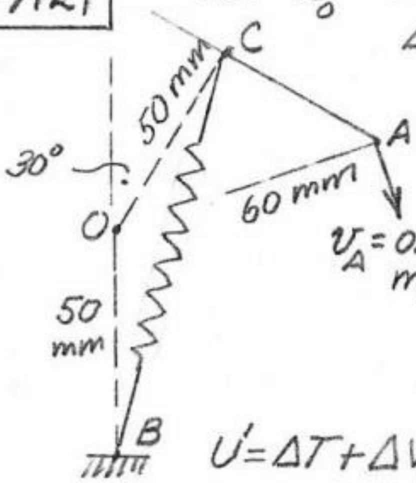
Let ℓ_o = unstretched length of spring



$\Delta V_e = \frac{1}{2}(1400)\left[(\overline{BC} - \ell_o)^2 - (0.10 - \ell_o)^2\right]$

$\overline{BC} = 2(0.05)\cos 15° = 0.0966$ m

$\Delta V_g = 0$ (horizontal plane)

$v_A = 0.25$ m/s

$\Delta T = \frac{1}{2} I_o \omega^2 = \frac{1}{2}(1.5)(0.055)^2\left(\frac{0.25}{0.06}\right)^2$

$= 0.0394$ ✓

$U' = 0$

$U' = \Delta T + \Delta V_e = 0$

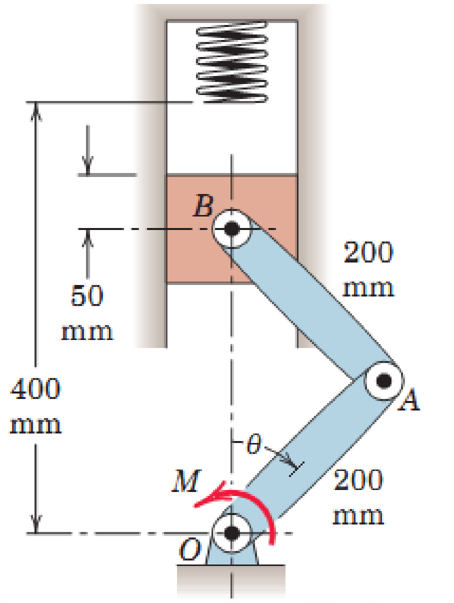
$0.0394 + 700\left[(0.0966 - \ell_o)^2 - (0.10 - \ell_o)^2\right] = 0$

$0.0394 - 0.4690 + 4.77\ell_o = 0, \quad \ell_o = 0.0900$ m

$\ell_o = 90.0$ mm

PROBLEM 6/135
(WORK-ENERGY)

Each of the two links has a mass of 2 kg and a centroidal radius of gyration of 60 mm. The slider at B has a mass of 3 kg and moves freely in the vertical guide. The spring has a stiffness of 6 kN/m. If a constant torque $M = 20$ N·m is applied to link OA through its shaft at O starting from the rest position at $\theta = 45°$, determine the angular velocity ω of OA when $\theta = 0$.



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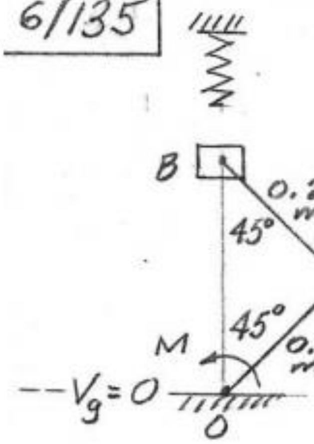
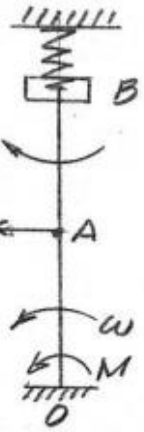




Link OA;

$\Delta V_g = 2(9.81)(0.1)(1-\cos 45°)$
$= 0.575$ J

$I_O = m(\bar{k}^2 + \bar{r}^2)$
$= 2([0.060]^2 + [0.1]^2)$
$= 0.0272$ kg·m^2

$\Delta T = \frac{1}{2} I_O \omega^2 = 0.0136\,\omega^2$

Link AB;

$\Delta V_g = 2(9.81)[0.3 - 0.3\cos 45°]$
$= 1.724$ J

$\Delta T = 0.0136\,\omega^2$ same as OA

Block B;

$\Delta V_g = 3(9.81)(0.4)(1-\cos 45°)$
$= 3.448$ J ; $\Delta T = 0$

$\Delta V_e = \frac{1}{2} k x^2$
$= \frac{1}{2} 6000 (0.050)^2$
$= 7.5$ J

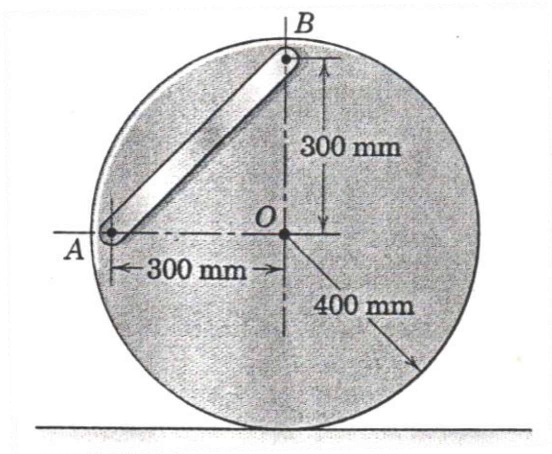
$U' = \int M\,d\theta = M\theta = 20\frac{\pi}{4}$
$= 15.71$ J

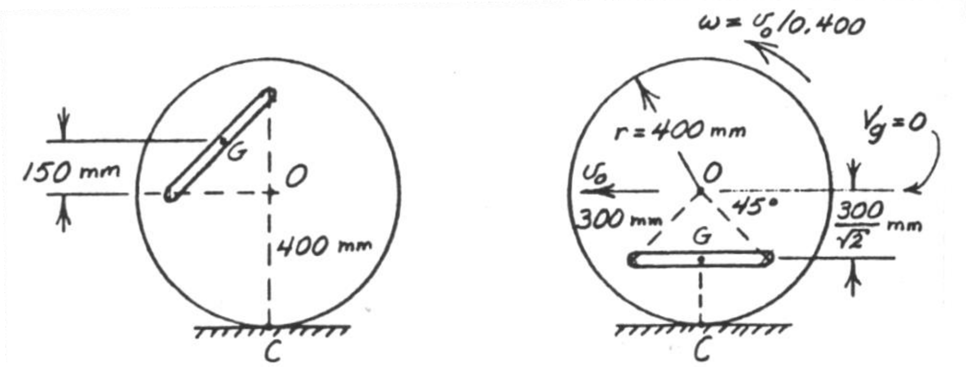
$U' = \Delta T + \Delta V_g + \Delta V_e$; $15.71 = 2(0.0136\,\omega^2) + (0.575 + 1.724 + 3.448) + 7.5$

$\omega^2 = 90.5$, $\omega = 9.51$ rad/s

PROBLEM 6/153 (6^{th} Ed.)
(WORK-ENERGY)

The 45-kg uniform circular disk with its attached 9-kg slender bar is released from rest in the position shown and rolls without slipping on the horizontal surface. Calculate the velocity v_O of the center O when the mass center of the bar is directly below the center O of the disk.





$v_G = \frac{\overline{CG}}{r} v_o$, $I_G = \frac{1}{12} 9 (0.300\sqrt{2})^2 = 0.135 \text{ kg}\cdot\text{m}^2$

$U'_{1-2} = 0 = \Delta T + \Delta V_g$ (1)

$\Delta T_{disk} = \frac{1}{2} 45 v_o^2 + \frac{1}{2}\left[\frac{1}{2} 45 (0.4)^2\right]\left(\frac{v_o}{0.4}\right)^2 - 0$

$= 33.8 v_o^2$

$\Delta T_{bar} = \frac{1}{2} m \bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2} 9\left[\frac{0.4 - 0.3/\sqrt{2}}{0.4} v_o\right]^2$

$+ \frac{1}{2} 0.135 \left(\frac{v_o}{0.4}\right)^2 - 0 = 1.415 v_o^2$

$\Delta V_g = -mgh = -9(9.81)(0.150 + 0.3/\sqrt{2}) = -32.0$ J

Eq. (1): $0 = (33.8 + 1.415) v_o^2 - 32.0$

$\underline{v_o = 0.954 \text{ m/s}}$