

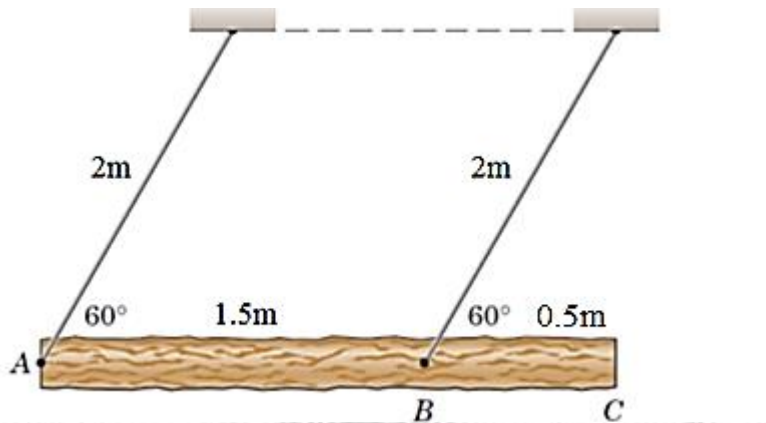
ME 206 – DYNAMICS – SPRING 2017

STUDY PROBLEMS-11

(PLANE KINETICS OF RIGID BODIES-6A.NEWTON'S LAW)

**PROBLEM 6/19
(CURV. TRANS.)**

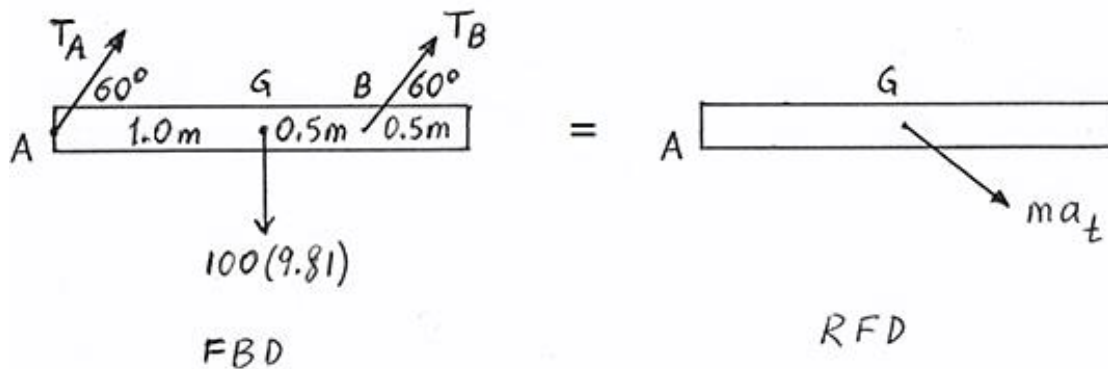
The uniform 100kg log is supported by the two cables and used as a battering ram. If the log is released from rest in the position shown, calculate the initial tension induced in each cable immediately after release and the corresponding angular acceleration α of the cables.



Curvilinear Translation : $\vec{a}_G = \vec{a}_A = \vec{a} = \vec{a}_t + \vec{a}_n$

$a_n = \omega^2 r = 0$ (ω is angular velocity of cable, r is cable length)

$a_t = \alpha r$, perpendicular to cable



$$\Sigma F_t = ma_t : 100(9.81) \cos 60^\circ = 100 a_t,$$

$$a_t = 4.90 \text{ m/s}^2$$

$$\alpha = a_t / r = 4.90 / 2 = \underline{2.45 \text{ rad/s}^2}$$

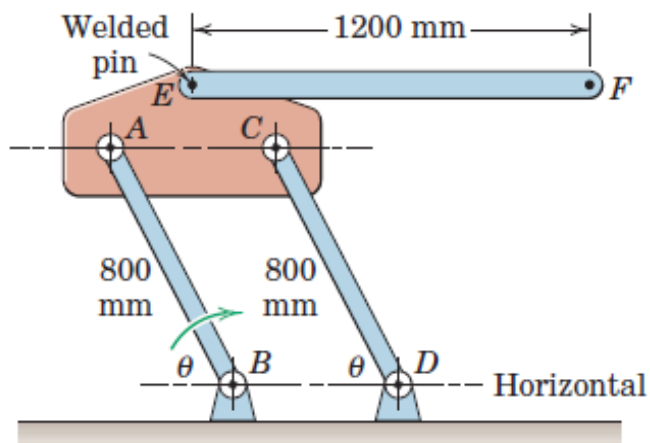
$$\downarrow + \Sigma M_G = 0 : T_B \sin 60^\circ \times 0.5 - T_A \sin 60^\circ \times 1.0 = 0, T_A = \frac{1}{2} T_B$$

$$\Sigma F_n = ma_n = 0 : T_A + T_B - 100(9.81) \sin 60^\circ = 0, T_A + T_B = 850 \text{ N}$$

$$\text{Combine \& get } \underline{T_A = 283 \text{ N}}, \underline{T_B = 567 \text{ N}}$$

PROBLEM 6/23
(CURV. TRANS.)

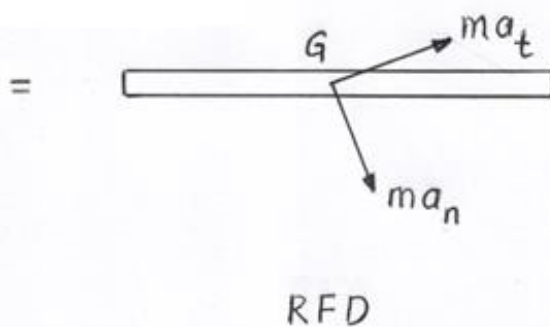
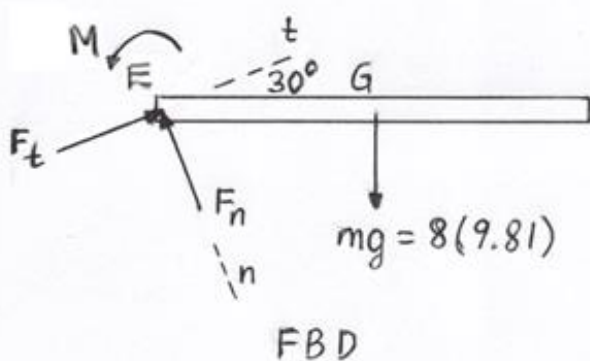
The parallelogram linkage shown moves in the vertical plane with the uniform 8-kg bar EF attached to the plate at E by a pin which is welded both to the plate and to the bar. A torque (not shown) is applied to link AB through its lower pin to drive the links in a clockwise direction. When θ reaches 60° , the links have an angular acceleration and an angular velocity of 6 rad/s^2 and 3 rad/s , respectively. For this instant calculate the magnitudes of the force F and torque M supported by the pin at E .



6/23 Curvilinear Translation: $a_E = a_A = a_C$

$$(a_E)_n = r\omega^2 = 0.8(3)^2 = 7.2 \text{ m/s}^2, \quad ma_n = 8(7.2) = 57.6 \text{ N}$$

$$(a_E)_t = r\alpha = 0.8(6) = 4.8 \text{ m/s}^2, \quad ma_t = 8(4.8) = 38.4 \text{ N}$$



$$\sum M_E = \sum mad : M - 8(9.81)(0.6) = 38.4(0.6 \sin 30^\circ) - 57.6(0.6 \cos 30^\circ), \quad M = 28.7 \text{ N}\cdot\text{m CCW}$$

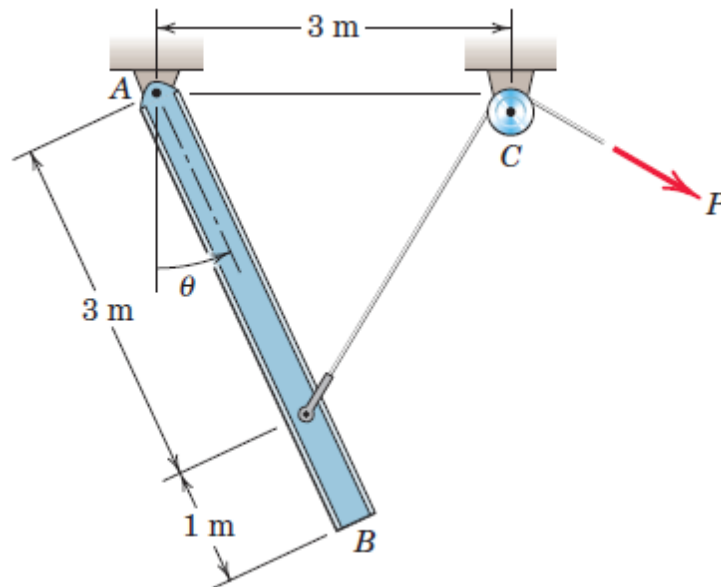
$$\sum F_t = ma_t : F_t - 8(9.81) \sin 30^\circ = 38.4, \quad F_t = 77.6 \text{ N}$$

$$\sum F_n = ma_n : -F_n + 8(9.81) \cos 30^\circ = 57.6, \quad F_n = 10.37 \text{ N}$$

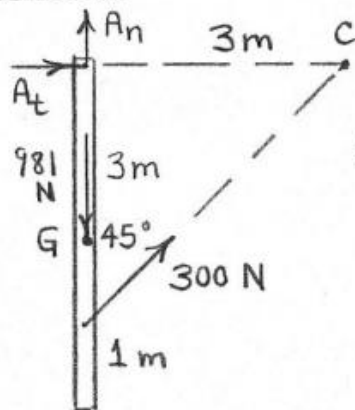
$$F = \sqrt{F_t^2 + F_n^2} = \underline{78.3 \text{ N}}$$

**PROBLEM 6/35
(FIXED AXIS ROT.)**

The uniform 100-kg beam is freely hinged about its upper end A and is initially at rest in the vertical position with $\theta = 0$. Determine the initial angular acceleration α of the beam and the magnitude F_A of the force supported by the pin at A due to the application of a force $P = 300$ N on the attached cable.



6/35



$$I_A = \frac{1}{3} m l^2 = \frac{1}{3} (100) (4)^2 = 533 \text{ kg}\cdot\text{m}^2$$

$$\sum M_A = I_A \alpha$$

$$300 (3 \sin 45^\circ) = 533 \alpha$$

$$\alpha = 1.193 \text{ rad/s}^2$$

$$\sum F_n = m \bar{a}_n : A_n + 300 \cos 45^\circ - 981 = 0, A_n = 769 \text{ N}$$

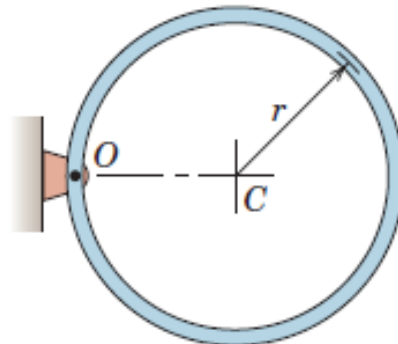
$$\sum F_t = m \bar{a}_t : A_t + 300 \sin 45^\circ = 100 (2) (1.193)$$

$$A_t = 26.5 \text{ N}$$

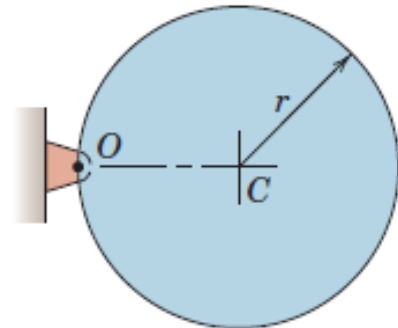
$$F_A = \sqrt{A_n^2 + A_t^2} = 769 \text{ N}$$

PROBLEM 6/38
(FIXED AXIS ROT.)

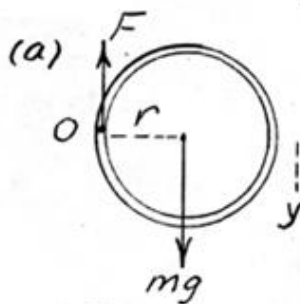
Determine the angular acceleration and the force on the bearing at O for (a) the narrow ring of mass m and (b) the flat circular disk of mass m immediately after each is released from rest in the vertical plane with OC horizontal.



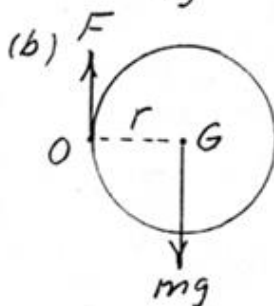
(a)



(b)



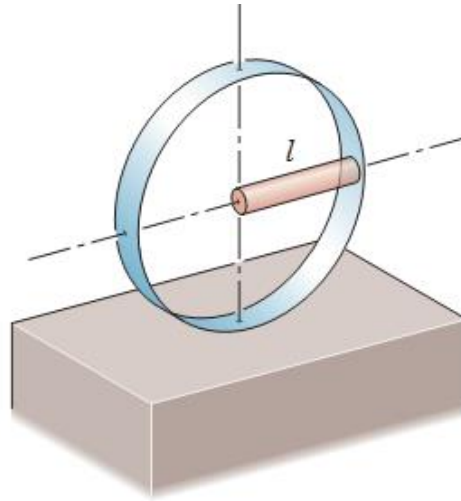
$$\begin{aligned} \Sigma M_O &= I_O \alpha; \quad I_O = I_G + mr^2; \quad I_G = mr^2 \\ mgr &= 2mr^2 \alpha \quad \alpha = \frac{g}{2r} \\ \Sigma F_y &= m\bar{a}_y; \quad mg - F = mr \left(\frac{g}{2r} \right) \\ &\underline{F = mg/2} \end{aligned}$$



$$\begin{aligned} \Sigma M_O &= I_O \alpha; \quad mgr = \left(\frac{1}{2}mr^2 + mr^2 \right) \alpha \\ &\underline{\alpha = \frac{2g}{3r}} \\ \Sigma F_y &= m\bar{a}_y; \quad mg - F = mr \left(\frac{2g}{3r} \right) \\ &\underline{F = mg/3} \end{aligned}$$

PROBLEM 6/83
(GEN. PLANE M.)

A uniform slender rod of length l and mass m is secured to a circular hoop of radius l as shown. The mass of the hoop is negligible. If the rod and hoop are released from rest on a horizontal surface in the position illustrated, determine the initial values of the friction force F and normal force N under the hoop if friction is sufficient to prevent slipping.

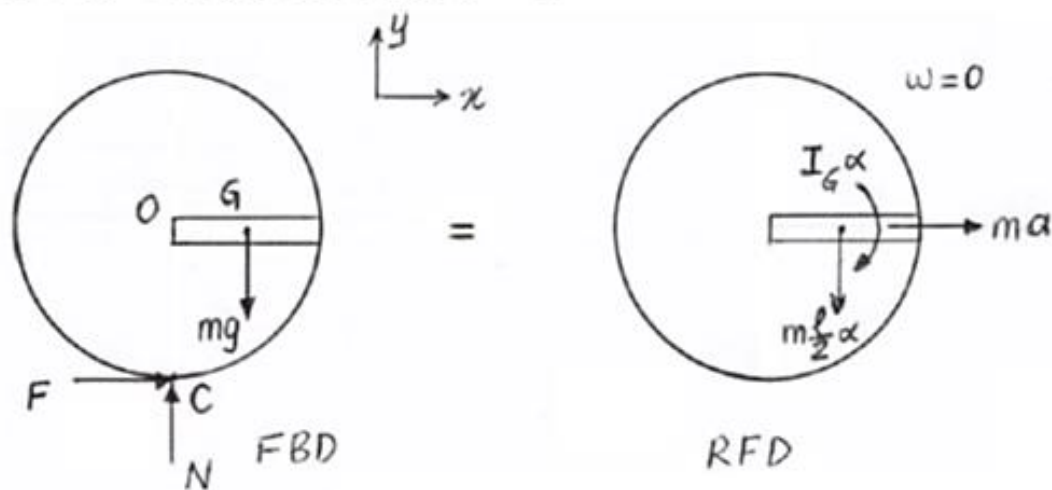


$$\vec{a}_G = \vec{a}_O + \vec{\alpha} \times \overline{OG} + \vec{\omega} \times (\vec{\omega} \times \overline{OG})$$

\vec{a}_O : magnitude is a , direction is along x

$\vec{\alpha} \times \overline{OG}$: magnitude is $\frac{\ell}{2} \alpha$, direction is perpendicular to OG

$\vec{\omega} \times (\vec{\omega} \times \overline{OG})$ is zero since $\omega = 0$.



No slip condition : $a = \ell \alpha$

$$\sum M_C = I_G \alpha + ma\ell + m \frac{\ell}{2} \alpha \frac{\ell}{2} \Rightarrow mg \frac{\ell}{2} = \frac{1}{12} m \ell^2 \frac{a}{\ell} + ma\ell + m \frac{\ell a \ell}{2 \ell 2}$$

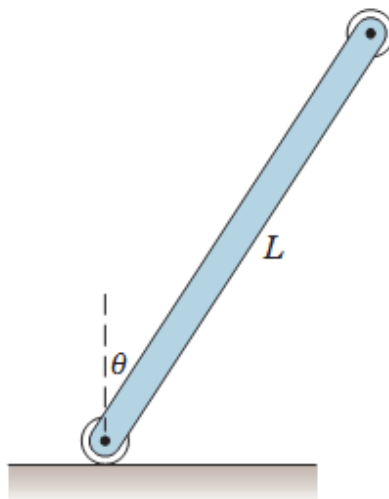
$$\Rightarrow a = \frac{3}{8} g, \quad \alpha = \frac{3g}{8\ell}$$

$$\sum F_x = ma_{Gx} \Rightarrow F = ma = \frac{3}{8} mg$$

$$\sum F_y = ma_{Gy} \Rightarrow N - mg = -m \frac{\ell}{2} \alpha \Rightarrow N = \frac{13}{16} mg$$

PROBLEM 6/104
(GEN. PLANE M.)

The uniform slender bar of mass m and length L with small end rollers is released from rest in the position shown with the lower roller in contact with the horizontal plane. Determine the normal force N under the lower roller and the angular acceleration α of the bar immediately after release.

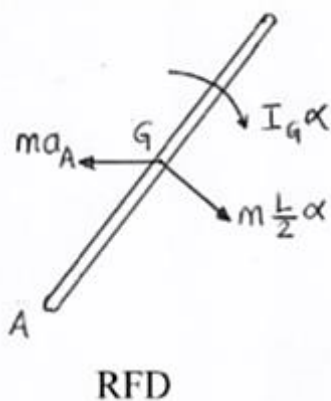
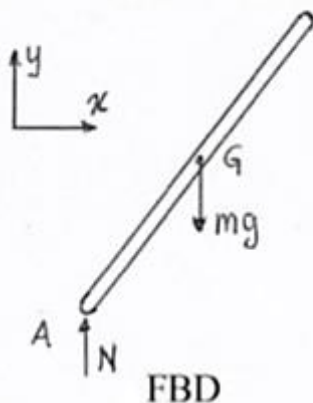


$$\vec{a}_G = \vec{a}_A + \vec{\alpha} \times \overline{AG} + \vec{\omega} \times (\vec{\omega} \times \overline{AG})$$

\vec{a}_A : magnitude is a_A , direction is along x (sense is arbitrarily taken towards left)

$\vec{\alpha} \times \overline{AG}$: magnitude is $\frac{L}{2}\alpha$, direction is perpendicular to AG (sense of α is arbitrarily taken as cw)

$\vec{\omega} \times (\vec{\omega} \times \overline{OG})$ is zero since $\omega = 0$.



$$\sum M_A = I_G \alpha - ma_A \frac{L}{2} \cos \theta + m \frac{L}{2} \alpha \frac{L}{2} \Rightarrow mg \frac{L}{2} \sin \theta = \frac{1}{12} mL^2 \alpha - ma_A \frac{L}{2} \cos \theta + m \frac{L^2}{4} \alpha \quad (1)$$

$$\sum F_x = ma_{Gx} \Rightarrow 0 = -ma_A + m \frac{L}{2} \alpha \cos \theta \quad (2)$$

From equations (1) and (2) one can find a_A and α . $\Rightarrow \alpha = \frac{2g \sin \theta}{L \frac{1}{3} + \sin^2 \theta}$

$$\sum F_y = ma_{Gy} \Rightarrow N - mg = -m \frac{L}{2} \alpha \sin \theta \Rightarrow N = mg - m \frac{L}{2} \frac{2g \sin \theta}{L \frac{1}{3} + \sin^2 \theta} \sin \theta = \frac{mg}{1 + 3 \sin^2 \theta}$$