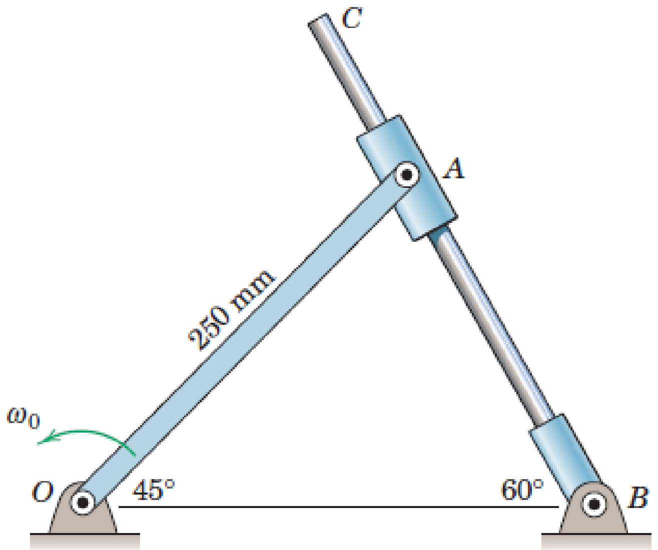
ME 206 – DYNAMICS – SPRING 2017
STUDY PROBLEMS-10 (RIGID BODY KINEMATICS-SECTION 5.7)

PROBLEM 5/169

Bar OA has a counterclockwise angular velocity $\omega_0 = 2$ rad/s. Rod BC slides freely through the pivoted collar attached to OA. Determine the angular velocity ω_{BC} of rod BC and the velocity of collar A relative to rod BC.



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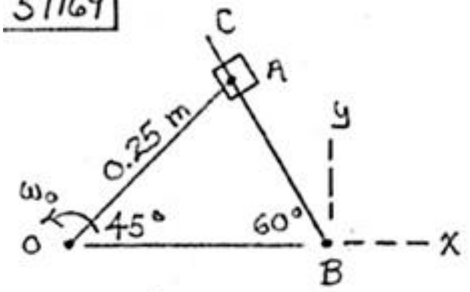


$$\frac{\sin 45^\circ}{\overline{AB}} = \frac{\sin 60^\circ}{0.25}$$

$$\overline{AB} = 0.204 \text{ m}$$

Use frame B_{xy} fixed to link BC

Eq. 5/12: $\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

$\underline{v}_A = \underline{\omega}_0 \times \underline{r}_{A/O} = 2\underline{k} \times 0.25(\cos 45^\circ \underline{i} + \sin 45^\circ \underline{j})$

$\underline{v}_B = 0$, $\underline{\omega} = \omega_{BC}\underline{k}$

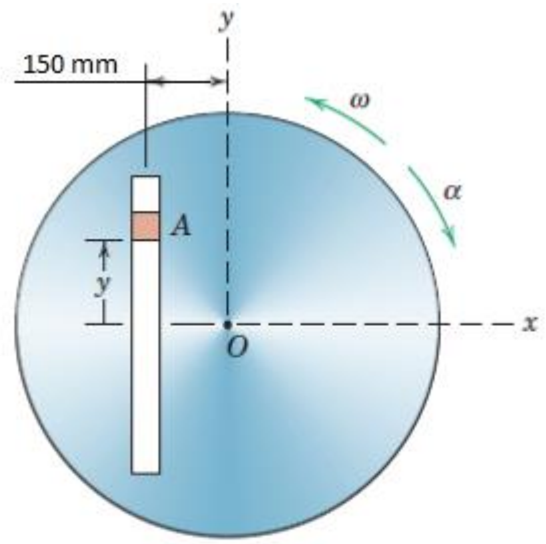
$\underline{r} = \overline{AB}(-\cos 60^\circ \underline{i} + \sin 60^\circ \underline{j})$

$\underline{v}_{rel} = v_{rel}(-\cos 60^\circ \underline{i} + \sin 60^\circ \underline{j})$

With the usual algebra, $\begin{cases} \omega_{BC} = 0.634 \frac{\text{rad}}{\text{s}} \text{ CCW} \\ v_{rel} = 0.483 \text{ m/s} \end{cases}$

PROBLEM 5/159

The disk rotates about a fixed axis through O with angular velocity $\omega = 5$ rad/sec and angular acceleration $\alpha = 3$ rad/sec^2 at the instant represented, in the directions shown. The slider A moves in the straight slot. Determine the absolute velocity and acceleration of A for the same instant, when $y = 250$ mm, $\dot{y} = -600$ mm/sec, and $\ddot{y} = 750$ mm/sec^2.



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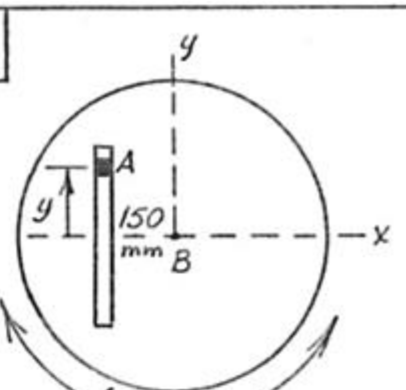


Attach Bxy to disk as shown. In Eqs. 5/12 & 5/14:

$\underline{v}_B = \underline{a}_B = \underline{0}$

$\underline{\omega} = 5\underline{k}$ rad/s, $\underline{\dot{\omega}} = -3\underline{k}$ rad/s^2

$\underline{r} = -150\underline{i} + 250\underline{j}$ mm

$\underline{v}_{rel} = -600\underline{j}$ mm/s

$\underline{a}_{rel} = 750\underline{j}$ mm/s^2

Eq. 5/12, $\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel}$, yields

$\underline{v}_A = -1250\underline{i} - 1350\underline{j}$ mm/s

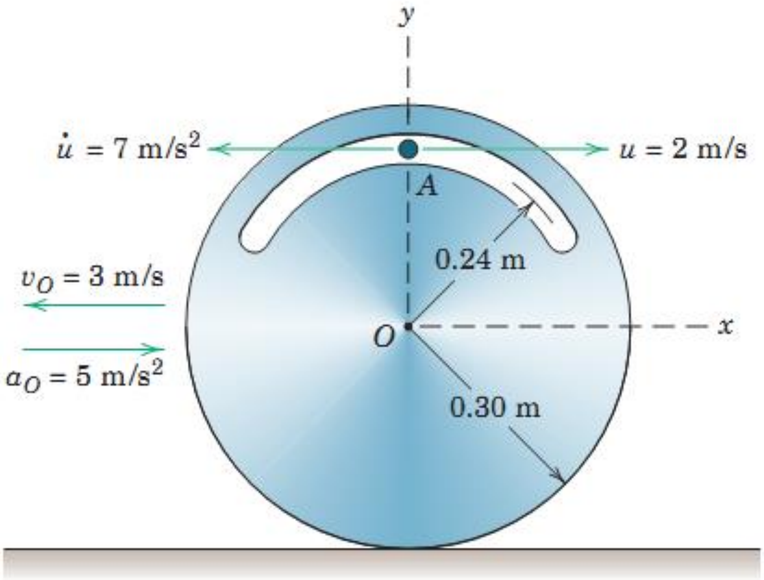
or $\underline{-1.25\underline{i} - 1.35\underline{j}}$ m/s

Eq. 5/14, $\underline{a}_A = \underline{a}_B + \underline{\alpha}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$,

yields $\underline{a}_A = 10.5\underline{i} - 5.05\underline{j}$ m/s^2

PROBLEM 5/162

The disk rolls without slipping on the horizontal surface, and at the instant represented, the center O has the velocity and acceleration shown in the figure. For this instant, the particle A has the indicated speed u and time rate of change of speed $\dot{u}$, both relative to the disk. Determine the absolute velocity and acceleration of particle A.



For the coordinates (y up, x right, + counterclockwise), the no-slip constraints are $v_0 = -r\omega$ & $a_0 = -r\alpha$. So

$$\omega = -\frac{v_0}{r} = -\frac{-3}{0.30} = 10 \text{ rad/s}$$

$$\alpha = -\frac{a_0}{r} = -\frac{5}{0.30} = -16.67 \text{ rad/s}^2$$

Use the frame Oxy as disk-fixed.

(5/12): $\underline{v}_A = \underline{v}_0 + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

(5/14): $\underline{a}_A = \underline{a}_0 + \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

Ingredients:
$\underline{v}_0 = -3\underline{i}$ m/s
$\underline{a}_0 = 5\underline{i}$ m/s²
$\underline{\omega} = 10\underline{k}$ rad/s
$\underline{\alpha} = -16.67\underline{k}$ rad/s²

$\underline{r} = 0.24\underline{j}$ m
$\underline{v}_{rel} = 2\underline{i}$ m/s
$\underline{a}_{rel} = -7\underline{i} - \frac{2^2}{0.24}\underline{j}$
$= -7\underline{i} - 16.67\underline{j}$ m/s²

Substitute into (5/12) & (5/14) & simplify:

$\underline{v}_A = -3.4\underline{i}$ m/s

$\underline{a}_A = 2\underline{i} - 0.667\underline{j}$ m/s²